Leonhard Euler
His Life, The Man, and His Work

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The three stations of Euler's life

Basel 1707-1727

St. Petersburg 1727–1741

Berlin 1741–1766

St. Petersburg 1766–1783

Basel 1707–1727

Auspicious beginnings

Chronology

- born April 15, 1707, the first of four children
- parents: Paulus Euler (1670–1745), a Protestant minister, and Margaretha Brucker (1677–1761)
- early childhood at the parish residence in Riehen near Basel
- at the age of 8, sent to the Latin school in Basel
- University of Basel 1720–1726
- 1726: participates in a prize question of the Paris Academy with a memoir on the optimal placing of masts on a ship
- 1727: applied for the physics chair at the university with a work on the theory of sound
- left Basel (for good) in April of 1727 to assume a junior appointment at the Academy of St. Petersburg

St. Petersburg 1727–1741

Meteoric rise to world fame and academic advancement

Chronology

- groundwork for Euler's appointment at the Academy had been laid by Johann Bernoulli and his sons Niklaus II and Daniel I, both already active at the Academy
- 1731: professor of physics; ordinary member of the Academy
- 1733: succeeds Daniel Bernoulli (who returns to Basel) as professor of mathematics

Major Works

Mechanica: Analytic theory of motion (1736)

Tentamen novae theoriae musicae: Music theory (1739)

Scientia Navalis: Naval science (1749, written 1740–41)

plus some 70 memoirs on

- analysis
- number theory
- physics and mechanics
- astronomy

Selecta Euleriana

Selectio 1 The Basel problem (1740)

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

zeta function

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots$$

Euler determines

$$\zeta(4),\ \zeta(6),\ \zeta(8), \ldots,\ \zeta(12)$$

Later in 1750, he was able to prove (rigorously)

$$\zeta(2n) = \frac{2^{2n-1}}{(2n)!}\,|B_{2n}|\pi^{2n}$$

Selectio 2 Prime numbers and the zeta function

$$\mathcal{P} = \{2, 3, 5, 7, 11, 13, 17, \ldots\}$$

product formula (1737)

$$\prod_{p \in \mathcal{P}} \frac{1}{1 - 1/p^s} = \zeta(s)$$

Euler's derivation

from $\zeta(s)$ "peel away" all terms divisible by 2

$$\left(1 - \frac{1}{2^s}\right) \zeta(s) = 1 + \frac{1}{3^s} + \frac{1}{5^s} + \frac{1}{7^s} + \cdots$$

from this do the same with the prime 3, then with 5, etc.

$$\prod_{p \in \mathcal{P}} \left(1 - \frac{1}{p^s}\right) \zeta(s) = 1 \qquad \square$$

Berlin 1741–1766

The emergence of epochal treatises

Chronology

- 1746: Berlin Academy opens its doors, with Maupertuis its president and Euler the director of the Mathematics Class; elected foreign member of the Royal Society of London
- 1750: Euler's widowed mother comes to Berlin, where she lives in Euler's country estate together with Euler's sister-in-law and children
- 1752: Maupertuis returns to Paris, distraught; Euler takes over the Academy as *de facto*, if not *de jure*, president
- 1755: elected foreign member of the Paris Academy
- 1763– : Euler's relationship with Frederick II sours
- 1766: Euler returns to St. Petersburg

Major Works

Methodus inveniendi lineas curvas (1744)

Cometary and planetary trajectories (1744)

Optics (1746)

Neue Grundsätze der Artillerie (1745)

Introduction to the analysis of the infinite (1748)

Differential calculus (1755)

Integral calculus (1763, 1773)

Theoria motus corporum solidorum seu rigidorum (1765)

Dioptrics (1769–1771)

Letters to a German princess (written 1760–1762)

plus some 280 memoirs on

- analysis
- physics and mechanics
- astronomy
- optics
- number theory
- miscellanea

Selecta Euleriana

Selectio 3 The Königsberg bridge problem (1741)

connected graph

- path
- circuit
- Eulerian path or circuit
- degree of a vertex

Theorem (Euler) Let n be the number of vertices of odd degree.
(a) If $n = 0$, the graph has at least one Eulerian circuit;
(b) if $n = 2$, it has at least one Eulerian path, but no circuit;
(c) if $n > 2$, it has neither.
($n = 1$ is impossible.)

Königsberg bridge graph: $n = 4$

Selectio 4 Euler flow (1757)

Transonic Euler flow at Mach .85 about a cylinder

Selectio 5 Euler's polyhedral formula (1753)

In a three-dimensional convex polyhedron let

$$V = \text{ number of vertices}$$
$$E = \text{ number of edges}$$
$$F = \text{ number of faces}$$

Theorem (Euler)

$$V - E + F = 2$$

St. Petersburg 1766–1783

The glorious final stretch

Chronology

- 1771: Euler loses his (good) left eye following a cataract operation and becomes virtually blind; Euler's wooden house burns down during the great St. Petersburg fire
- 1773: Euler's wife Katharina dies
- 1776: Euler remarries
- 1783: On September 18, Euler dies of a stroke

Major Works

Vollständige Anleitung zur Algebra (1770)

Second lunar theory (1772)

Second theory of ships (1773)

plus some 450 memoirs on

- analysis
- geometry
- number theory
- mechanics
- astronomy
- optics
- miscellaneous

Selecta Euleriana

Selectio 6 Euler's disk

Selectio 8 Gear transmission; Euler's tooth profile

The Man

Personality

- modest, inconspicuous, uncomplicated, yet cheerful and sociable
- "honesty, uncompromising rectitude—the acknowledged national virtues of Swiss people—he possessed to a superior degree" (Fuss)
- free of priority concerns
- generous in acknowledging and furthering other people's work

Intellect

- phenomenal memory, erudite
- unusual power of mental calculation
- ability to concentrate on mental work under adverse conditions

Craftsmanship

- superb expositor
- his goal: ultimate clarity and simplicity
- yet fearless and aggressive in his quest for discovery

Epilogue

LEONHARD EULER
1707–1783
mathematician, physicist, engineer, astronomer and philosopher, spent his youth in Riehen. He was a great scholar and a kind man.