Problemath series 4

1 February 2010

Problemath 11

Midnight chimed in London. Doctor Watson, who was immerged in the reading of an old algebra book, suddenly exclaimed:"Good Lord! This system of linear equations, which has a unique solution, truly shows a remarkable property: if the system is written with matrices $AX = B$, then the coefficients

$$a_{11}, a_{12}, ..., a_{1n}, b_{1,} a_{21}, a_{22}, ..., b_{2,} ... a_{m1}, a_{m2}, ... a_{mn}, b_m$$

Create an arithmetic sequence of non-zero common difference.

Sherlock Holmes looked up, interested: "Looking at the thickness of the dust layer covering this old math book of yours, Dear Watson, I can certainly deduce that this system of equations has real coefficients, and is made of at least two equations."

The doctor just had time to confirm these hypotheses when the famous detective handed him a piece of paper on which he just scribbled numbers.

Amazing ! Holmes, your are amazing! How on Earth could you find the solution of this system without having seen it and without even knowing the number of equations or unknowns?"

"Elementary, my Dear Watson!" and Sherlock Holmes puffed his pipe.

How did Sherlock Holmes reach the solution, and what is the solution?

Problemath 12

The real number r is said to be <u>inevitable</u> if, for all continuous function $f : [0,1] \to \mathbb{R}$ such that $f(0) = f(1) = 0$, the graph of f possesses an " horizontal chord" of length r, that is , if there exists $x \in [0,1]$ such that $x + r \in [0,1]$ and $f(x) = f(x + r)$. The number 1 is clearly an inevitable number. For which values of the whole number $n > 1$ is the number $\frac{1}{n}$ inevitable?

Problemath 13

Saint Michael (who, as everyone knows is the symbol of Brussels) once again battles the dragon. This time the beast has 99 heads and 99 tails. Piece of cake! Michael's magic sword comes in action! One blow can chop off either one head or two heads or one tail or two tails.

If only one head is chopped off, then it immediately grows back again. If only one tail is cut, then it is replaced by two new tails. If once single blow cuts two tails, then one new head appears, and if one single blow chops off two heads at once, then nothing comes back!

What is the smallest number of blows Saint Michael will need to kill the dragon (that is, to chop off all its heads and all its tails)?

The solutions should be sent to: jdoyen@ulb.ac.be by Friday 26 february 2010, 14:00.