

**Year 2010-2011**

**Problemath Series 1**

**20 September 2010**

(Deadline: Friday 8 October 14:00)

**Problemath 1**

$abc$  is right triangle with the right angle at point  $a$ . Let  $p$  be a point on side  $[a,b]$ , and  $q$  be a point on side  $[a,c]$ . Given that  $|bc|=3$ ,  $|ap|=|aq|=1$ , and that there exists a point  $r$  of the hypotenuse  $[b,c]$  such that  $aprq$  is a square, what is the exact measure of angle  $\widehat{acb}$ ?

**Problemath 2**

A large  $3 \times 3 \times 3$  cube is made of twenty seven  $1 \times 1 \times 1$  small cubes the faces of which are all white. The faces of the large cube are painted in black, after which the cube is entirely dismantled. A blindfolded person randomly rebuilds the large cube out of its 27 smaller cubes. What is the probability that this large cube has all its faces black?

**Problemath 3**

$P(x)$  is a 2009<sup>th</sup> degree polynomial with real coefficients, such that  $P(n) = \frac{n}{n+1}$  for all whole numbers  $n \in \{0, 1, 2, \dots, 2009\}$ .

Evaluate  $P(2010)$ .